AMES/GRANT-CR
IN-34 56212
6P.

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Selection Principles and Pattern Formation in Fluid Mechanics and Nonlinear Shell Theory

Semiannual Progress Report, September 1, 1986 to February 28, 1987

*NASA-Ames NAG 2-278**

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(NASA-CR-180159) SELECTION PRINCIPLES AND PATTERN FORMATION IN FLUID MECHANICS AND NONLINEAR SHELL THEORY Semiannual Progress Report, 1 Sep. 1986 - 28 Feb. 1987 (Colorado Univ.) 6 p
N87-17004
Unclas
CSCL 20D G3/34 44000

*The NASA Technical Officer for this grant is Gary T. Chapman, NASA-Ames Research Center, Moffett Field, CA 94035.

Semiannual Progress Report NAG 2–278: September 1, 1986 to February 28, 1987.

The major research effort of the Principal Investigator during the indicated period again has been devoted to a study of vortex breakdown and this work is described in part I. Some work on spiral flows is reported on in part II and some comments on recent publications are included in part III.

I. *Wave theories of vortex breakdown*. The investigation has focused primarily on wave theories of vortex breakdown in a tube such as those in [1; 2; 6; 11; 12]. The approach described below is not necessarily restricted to weakly nonlinear waves so that it can be used to study "large" vortex breakdowns. A more formal approach was described in the last Progress Report, March 1, 1986 to August 31, 1986, and the approach there was compared with that of Benjamin in [1; 2]. Such formal approaches lead to qualitative results about solitary waves, however, they do not seem to lead to either a description of the actual mechanism of vortex breakdown or the determination of the parameter values at which vortex breakdown occurs. Perhaps a better setting for a more detailed investigation is that described below, a setting which involves dynamical systems and bifurcation of homoclinic and heteroclinic orbits in infinite-dimensional spaces.

We consider as in [1; 2] a swirl velocity, V, and an axial velocity, W, depending only on r, where r is the normalized radial coordinate of the tube, i.e., $0 < r < 1$ and $-\infty < x < \infty$. Such V and W determine a primary cylindrical flow with stream function ψ_0. The determination of axisymmetric inviscid flows bifurcating from the primary flow leads to the study of a system of ordinary differential equations of the form

$$(1) \qquad \frac{d\alpha}{dx} = a_1\beta + b_{11}\alpha\beta + c_{21}\alpha^2\beta + c_{03}\beta^3 + A_1(\alpha,\beta,\lambda)\,,$$

$$\frac{d\beta}{dx} = a_2\alpha + b_{02}\beta^2 + b_{20}\alpha^2 + c_{12}\alpha\beta^2 + c_{30}\alpha^3 + A_2(\alpha,\beta,\lambda)\,,$$

where the coefficients a_i, b_{ij} and c_{ij} depend upon a real parameter λ, the terms $A_i(\alpha,\beta,\lambda)$ are "higher order" in some appropriate sense, and $\alpha = \beta = 0$ corresponds to the given primary flow. The parameter λ varies near λ_c, the value of λ for which the primary flow is "critical" (e.g., see [1; 2; 11]); if $\lambda < \lambda_c$ the primary flow is "supercritical" and if $\lambda > \lambda_c$, then the primary flow is "subcritical".

E.g., if $W = \mu$ is a uniform axial flow and if V is of the form $V(r) = \gamma\, V_0(r)$, then $\lambda = \frac{\gamma}{\mu}$. Moreover, if $\Gamma = rV_0(r)$ is the circulation and if $y = \frac{1}{2}\, r^2$, then the determination of disturbance flows leads to the study of the nonlinear elliptic equation (e.g., see [1; 2])

$$(2) \qquad \frac{\partial^2\phi}{\partial x^2} + 2y\,\frac{\partial^2\phi}{\partial y^2} = -\lambda B(y-\phi)\phi, \quad 0 < y < 1,\ -\infty < x < \infty\,,$$

where $B(s) = \frac{\Gamma(s)\Gamma'(s)}{s}$. The key observations now are that if one writes (2) as a first order system with $u = \phi$ and $v = \frac{\partial\phi}{\partial x}$, then (a) the resultant system is "reversible" and (b) for λ near λ_c, the spectrum of the linearized problem has two real eigenvalues for $\lambda < \lambda_c$ that pass through 0 at $\lambda = \lambda_c$ and become imaginary for $\lambda > \lambda_c$. Thus, standard splitting methods from bifurcation theory (e.g., see [3]) can be used to derive a system of the form (1) in which for convenience the dependence upon a third infinite-dimensional component has been incorporated as part of the "higher order" terms. Systems of the form (1) can be derived in this way for a wide class of swirl velocities, V, and axial velocities, W.

By introducing various scalings and solving (1) in these special cases, one obtains a variety of solutions including homoclinic and heteroclinic orbits. Since in the setting of dynamical systems homoclinic orbits may be considered as solitary waves, such an approach yields, in particular, the formal results in [2; 11] on solitary waves. E.g., by solving the system

$$\text{(3)} \qquad \frac{d\alpha}{dx} = a\beta + O(|\lambda-\lambda_c|) ,$$

$$\frac{d\beta}{dx} = b\alpha + c\alpha^2 + O(|\lambda-\lambda_c|) ,$$

where $a > 0, b > 0$ and $c \neq 0$ are real constants, one sees that, if $\lambda < \lambda_c$, then $(\alpha,\beta) = (0,0)$ is a saddle point connected to itself by a separatrix. Thus, there is a solitary wave bifurcating "to the left" at $\lambda = \lambda_c$ from the primary flow.

A more interesting situation that seems to provide a mechanism for describing "large" vortex breakdowns is described by the scaling leading to the system

$$\text{(4)} \qquad \frac{d\alpha}{dx} = a\beta + O(|\lambda-\lambda_c|) ,$$

$$\frac{d\beta}{dx} = b\alpha + c\alpha^2 + d\alpha^3 + O(|\lambda-\lambda_c|) ,$$

where a, b, c and d are constants. One now can show that for certain values of λ there are two saddle points of (4) connected to each other by trajectories; since one of the saddle points corresponds to (0,0), i.e., the primary flow, we have the possibility of "large" amplitude wave-like transition solutions between the primary flow and a second state of the physical system. The existence of such "large" amplitude transition solutions would remove one of the basic objections to the weakly nonlinear wave theories in [1; 2; 11], namely it was postulated in [1; 2] that certain large amplitude transition waves of unknown structure would have to be present in the physical system but there was no explanation given as to how "small" perturbations might lead to "large" amplitude transition solutions.

The qualitative results described above depend upon various generic properties of the coefficients in (1), (3) or (4) and at present the investigator has not been able to verify such properties except in the case of (3). Work continues to determine, in particular, the coefficients in (4) and to develop a theory of "large" amplitude transition solutions.

II. *Spiral states in rotating flows*. The problem of rotating plane Couette flow has been solved by means of the structure parameter approach and the final version of the paper [10] is being prepared (with G.H. Knightly). If one uses the set up described in [7, §51 ff.], then the appropriate structure parameter is proportional to $\sin(\chi - \psi)$, where χ is the spiral angle of the basic flow and ψ is the spiral angle of the disturbance flow. Using such a structure parameter one can show the somewhat surprising result that the problem has the same operator formulation as the much simpler problem of combined Couette-Poiseuille channel flow treated in [9]. The results obtained for plane Couette flow justify many of the formal calculations in [7] and apparently provide the first detailed bifurcation results for viscous spiral flows; in particular, such results show that a vanishingly small axial shear would be sufficient to destabilize a (pure) swirling flow, a result obtained previously in [13] by means of a formal inviscid analysis.

III. *Publications*. The papers [8] and [9] are now in press and the paper [14] on the Taylor problem for "short" cylinders has been accepted for publication. The qualitative analytical results in [14] seem to complement the striking numerical results in [4; 5]. E.g., what we have called a stable 1-cell flow in [14] actually has a "weak" 2-cell component as well and this is in close agreement with the single-cell flow shown in [4, Figure 4(c)] except that the streamline patterns are, of course, determined in detail in [4]. On the other hand, some of the qualitative results in [14], such as the role of the stable (1+2)-cell flow and the reinstatement of the 2-cell flow as a stable stationary flow, do not seem to be obtained directly in the detailed numerical investigation in [4]. Roughly speaking, only about one-half of the qualitative solutions obtained in [14] appear to be determined at present by the numerical methods in [4].

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